А.Ш. Кажикенова (доцент), Д.Б. Алибиев (доцент),

Р.А. Оразбекова (преподаватель), А.С. Смаилова (преподаватель),

Уразымбетова Т.Ж. (М-19-1)

Карагандинский государственный университет им. Е.А. Букетова

Differential equations in economic problems

It is called a differential equation in which the unknown is a function of one or more variables, and in the equation includes not only the function but also its derivatives.

If the unknown function is a function of one independent variable, then the equation is called an ordinary differential, if - a function of several variables, - differential equations in partial derivatives.

The order of the highest derivative (or differential) included in the differential equation is called the order of the equation. In general, the ordinary differential equation n-th order can be written as:

$$F\left(x, y, y', y'', \dots, y^{(n)}\right) = 0.$$

Any function that satisfies the differential equation, called the solution or the integral of this equation.

The solution of the differential equation (if it exists), in which the number of arbitrary constants equal to the order of the equation, called the general solution of this differential equation.

The general solution of ordinary differential equations n th order is written as:

$$y = \varphi(x, C_1, C_2, \dots, C_n).$$

differential equation solutions for certain values of the arbitrary constants is called a particular solution. The conditions to be met by the required particular solution of the differential equation are called the initial conditions. The problem of finding a specific partial solution of this differential equation is called the initial data of the Cauchy problem. Since each particular solution of this differential equation is a function of one variable, it corresponds to a certain line in a rectangular coordinate system on the plane with this decision. It is called the integral curve of the differential equation. General solution of the differential equations corresponding to a plurality of all the integral curves of this equation, which is called a family of integral differential equation curves.

First order differential equation is an equation of the form: $F(x, y, y')=0$ or $y'=f(x, y)$. Its general solution contains one arbitrary constant *C:* $y=\varphi(x, C)$.

First order differential equation:

$$y'+p(x)y=f(x)$$

It called linear if it is linear in the unknown function y and its derivative y' where $p(x)\neq 0$ and $f(x)\neq 0$ - continuous function of x. The linear first order differential equation is homogeneous if $f(x)=0$, otherwise it is inhomogeneous. Linear differential equation can be integrated by Bernoulli, whose essence lies in the following. We represent the unknown function y as a product of two unknown functions $u(x)$ and $\upsilon(x)$ according to the formula $y=u(x)\upsilon(x)$ (Bernoulli substitution). Then $y'=u'\upsilon+u\upsilon'$. Substituting the expressions for y and y' in a linear differential equation, we get:

$$u'\upsilon+u\upsilon'+p(x)u\upsilon=f(x).$$

which convert to the form:

$$u(\upsilon'+p(x)\upsilon)+u'\upsilon=f(x).$$

Because $u\neq 0$, the integration of this type of equation is reduced to the integration of two equations with multiple variables:

$$\upsilon'+p(x)\upsilon=0 \text{ and } u'\upsilon=f(x).$$

Consider the examples of the application of differential equations in the economic dynamics. In the model of natural growth is believed that the rate of output (acceleration) is proportional to the magnitude of the investment, ie, $y'(t)=lI(t)$. Since the value of investments $I(t)$ is a fixed part of the income,

$I(t) = mY(t) = mpy(t)$, where m is the norm of investment. Thus, given this equation, the pattern of growth in a competitive market will look $y' = mlp(y)y$.

Example. Find an expression for the volume of sales $y = y(t)$, if it is known that the demand curve $p(y)$ It is given by the equation $p(y) = 2 - y$, The rate of acceleration $l = 2$, investment rate $m = 0{,}5$, $p(0) = 0{,}5$.

Solution. Growth pattern in this case takes the form

$$y' = mlp(y)y = (2 - y)y$$

or

$$\frac{dy}{(2-y)y} = dt.$$

Performing the integration term by term, we have

$$\ln\left|\frac{2-y}{y}\right| = -2t + C_1$$

or

$$\frac{2-y}{y} = Ce^{-2t}.$$

Because

$p(0) = 0{,}5$, the C = -3.

There fore

$$y = \frac{2}{1+3e^{-2t}}.$$