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Systems of the linear algebraic equations and inequalities

Let's consider application of matrixes and determinants for a research and the solution of a system of three equations of the first degree:

$$\begin{cases} a_1x+b_1y+c_1z=h_1 \\ a_2x+b_2y+c_2z=h \\ a_3x+b_3y+c_3z=h \end{cases}$$

Coefficients $a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3$ and free terms h_1, h_2, h_3 assumed to be given. Three-number set x_0, y_0, z_0 it is called the solution of the system (3), if the result of substitution of these numbers instead of x, y, z all three equations (3) turn into identities. The main role is played by the following four solutions:

$$\Delta=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix},\ \Delta_x=\begin{vmatrix} h_1 & b_1 & c_1 \\ h_2 & b_2 & c_2 \\ h_3 & b_3 & c_3 \end{vmatrix},\quad \Delta_y=\begin{vmatrix} a_1 & h_1 & c_1 \\ a_2 & h_2 & c_2 \\ a_3 & h_3 & c_3 \end{vmatrix},\ \Delta_z=\begin{vmatrix} a_1 & b_1 & h_1 \\ a_2 & b_2 & h_2 \\ a_3 & b_3 & h_3 \end{vmatrix}.$$

The solution Δ is called the determinant of the system (3). Determinants Δ_x, Δ_y, Δ_z are obtained from the determinant Δ replacement free members of the elements respectively first, second and third columns.

The following cases are possible.

Case 1 ($\Delta \neq 0$). In this case, there is only one solution to the system, and it can be found by the following formulas, which are called Cramer formulas:

$$x=\frac{\Delta_x}{\Delta};\ y=\frac{\Delta_y}{\Delta};\ z=\frac{\Delta_z}{\Delta}.$$

Case 2 ($\Delta=0$). In this case, the system solution may not exist or the system may have an infinite number of solutions. For example, the system

$$\begin{cases} x+y & =1 \\ x+y & =2 \end{cases}$$

has no solution and the system

$$\begin{cases} x+y & =1 \\ 2x+2y & =2 \end{cases}$$

has an infinite number of solutions.

The system of equations can also be solved by the Gauss method. The Gauss method consists in sequentially eliminating the unknowns and reducing the system to a stepwise form.

Example. The shoe factory specializes on release of products of three types: boot, sneaker and boot; at the same time raw materials of three S_1, S_2, S_3 types are used. Consumption rates of each of them on one pair of shoes and volume of a consumption of raw materials for 1 day are set by table 1.

Table 1 - flow Rates

Type of raw materials	Raw materials consumption rates on one couple, conventional unit.			Raw materials consumption for 1 day, conventional unit.
	wellingtons	sneakers	boots	
S_1	5	3	4	2700
S_2	2	1	1	900
S_3	3	2	2	1600

To find the daily volume of release of each type of footwear.

Solution. Let daily the factory exhaust h_1 a boot, h_2 pairs of sneakers and h3 pairs of boots.

Then we have the system of the equations:

$$\begin{cases} 5x_1+3x_2+4x_3=2700 \\ 2x_1+\ \ x_2+\ \ x_3=900 \\ 3x_1+2x_2+2x_3=1600 \end{cases}.$$

$$\Delta=\begin{vmatrix} 5 & 3 & 4 \\ 2 & 1 & 1 \\ 3 & 2 & 2 \end{vmatrix}=10+16+9-12-10-12=1;$$

$$\Delta_1=\begin{vmatrix} 2700 & 3 & 4 \\ 900 & 1 & 1 \\ 1600 & 2 & 2 \end{vmatrix}=5400+7200+4800-6400-5400-5400=200;$$

$$\Delta_2 = \begin{vmatrix} 5 & 2700 & 4 \\ 2 & 900 & 1 \\ 3 & 1600 & 2 \end{vmatrix} = 9000 + 12800 + 8100 - 10800 - 8000 - 10800 = 300;$$

$$\Delta_3 = \begin{vmatrix} 5 & 3 & 2700 \\ 2 & 1 & 900 \\ 2 & 2 & 1600 \end{vmatrix} = 8000 + 10800 + 5400 - 5400 - 9000 - 9600 = 200.$$

$$x_1 = \frac{\Delta_1}{\Delta} = 200,\ x_2 = \frac{\Delta_2}{\Delta} = 300,\ \ x_3 = \frac{\Delta_3}{\Delta} = 200.$$

Solving system different modes, we find out that the factory exhausts 200 pairs of boots, 300 pairs of sneakers, 200 pairs of boots.

Example. From two plants cars for two farms which requirements respectively 200 and 300 cars are delivered. The first plant has exhausted 350 cars, and the second - 150 cars. Costs of transportation of cars from the plant in each motor transport service (table 2) are known.

Table 2 - Costs of transportation

factory	Costs of transportation in motor transport service,	
	1	2
1	15	20
2	8	25

The minimum cost of transportation is equal to 7950 dollars the optimal plan of transportation of cars.

Solution. Let x_{ij} be the number of vehicles supplied from the *i*-th automobile plant (*i*,*j*=1,2). Get the system

$$\begin{cases} x_{11} + x_{12} = 350 \\ x_{21} + x_{22} = 150 \\ x_{11} + x_{21} = 200 \\ x_{12} + x_{22} = 300 \\ 15x_{11} + 20x_{12} + 8x_{21} + 25x_{22} = 7950 \end{cases}$$

Having solved the system by Gauss method, we obtain a solution (50,300,150,0).