А.Ш. Кажикенова (доцент), Д.Б. Алибиев (доцент),

Р.А. Оразбекова (преподаватель), Т.Ж. Уразымбетова (М-19-1)

Карагандинский университет им. Е.А. Букетова

Formulation of the optimization problem

1. ***The search strategy***. Presentation of methods for solving optimization problems start with the problem of minimization of functions of one variable. With the objectives of minimizing the one variable you have encountered when studying the initial chapters of mathematical analysis. We will study numerical methods for solving problems of minimization.

Statement of the problem. You want to find the minimum point and minimum value of the function $J(u)$, i.e. a point $u^* \in R$ such that

$$J(u^*) = \min_{u \in R} J(u).$$

Highlight the class of functions in which all points of local minimum points are global minimum.

Definition 1. Function $J(u)$ is called *unimodal* on the interval $U = \{u : a \le u \le b\} = [a,b]$, if it is continuous on $[a,b]$ and there are numbers $\alpha,\ \beta,\ a \le \alpha \le \beta \le b$, such that

1) $J(u)$ strictly monotonically decreasing when $a \le u \le \alpha$ (if $a < \alpha$);

2) $J(u)$ strictly monotonically increasing when $\beta \le u \le b$ (if $\beta < b$);

3) $J(u) = J_* = \inf_{u \in U} J(u)$ when $\alpha \le u \le \beta$, so that $U_* = [\alpha, \beta]$. Cases when one or two of the segments $[a,\alpha],\ [\alpha,\beta],\ [\beta,b]$ degenerate to a point, are not excluded. In particular, if $\alpha = \beta$, then $J(u)$ is called *strictly unimodal* on the interval $[a,b]$.

In optimization problems there are two fundamentally different strategies of choosing points at which to calculate the function. If all points are specified in advance, before computing, is a *passive strategy*. If points are selected sequentially in the search process, taking into account the results of previous calculations, is *coherent strategy*. A coherent strategy can be implemented in the following ways:

a) use quadratic and cubic interpolation, where several of the computed function values is constructed interpolation polynomial, and its minimum indicates the next approximation of the desired extreme points;

b) building a sequence of nested sub-intervals, each of which contains the minimum point. The *search strategy* includes:

1. The choice of the initial interval. The boundaries of the interval of uncertainty $[a_0,\ b_0]$ must be such that function $J(u)$ was unimodal.

2. Reducing the interval of uncertainty

3. The test end conditions. The search ends when the length of the current interval of uncertainty $[a_k,\ b_k]$ is less than the set value.

2. The Swenn Algorithm. For the heuristic choice of the initial interval of uncertainty it is possible to apply the Swenn algorithm:

1. To arbitrarily specify the following parameters: u^0 - some starting point, $t > 0$ - size of step. Set $k = 0$.

2. Calculate the value of the function $J(u)$ in three points:

$$u^0 - t,\ u^0,\ u^0 + t.$$

3. Check the ending condition:

a) If $J(u^0 - t) \geq J(u^0) \leq J(u^0 + t)$, then the initial interval of uncertainty was found:

$$[a_0,\ b_0] = [u^0 - t,\ u^0 + t].$$

b) If $J(u^0 - t) \leq J(u^0) \geq J(u^0 + t)$, then function is not unimodal, and desired interval of uncertainty may not be found. The computation at this stops (it is recommended to specify a different starting point u^0).

c) If the ending condition is not satisfied, then go to step 4.

4. Determine the amount Δ:

a) If $J(u^0 - t) \geq J(u^0) \geq J(u^0 + t)$, then $\Delta = t;\ a_0 = u^0,\ u^1 = u^0 + t,\ k = 1$.

b) If $J(u^0 - t) \leq J(u^0) \leq J(u^0 + t)$, then $\Delta = -t;\ b_0 = u^0,\ u^1 = u^0 - t,\ k = 1$.

5. Find next point $u^{k+1} = u^k + 2^k \Delta$.

6. Check condition of decreasing function:

a) If $J(u^{k+1}) < J(u^k)$ and $\Delta = t$, then $a_0 = u^k$. If $J(u^{k+1}) < J(u^k)$ and $\Delta = -t$, then $b_0 = u^k$. In both cases set $k = k+1$ and go to step 5.

b) If $J(u^{k+1}) \geq J(u^k)$, procedure ends.

At $\Delta = t$ set $b_0 = u^{k+1}$, and at $\Delta = -t$ set $a_0 = u^{k+1}$. In result we have $[a_0, b_0]$ - the desired uncertainty interval.

Further, reducing the interval of uncertainty is carried out using a sequential strategy, i.e. is based on calculating the function at two points of the current interval. Property unimodality allows you to determine which of the possible subintervals the minimum was missing. As a new interval does the interval certainly containing the minimum point (method the halving method the Golden section method, Fibonacci).

Then evaluate the efficiency of algorithms for reducing the interval of uncertainty for a given number N of function evaluations, we introduce the criterion.

Defenition 2: Characteristics $R(N)$ relative reduction of the initial uncertainty interval is the ratio

$$R(N) = \frac{|L_N|}{|L_0|},$$

where $|L_N|$ - the length of the interval, as a result N of function evaluations, $|L_0|$ - the length of the initial interval of uncertainty.

3. Method of sorting. This method applies to passive strategies search points minimum. Describe the algorithm of brute force method for solving problems of one-dimensional minimization:

1. Let $[a_0, b_0]$ - the initial interval of uncertainty. Calculate points, equally spaced from each other by the formula $u_i = a_0 + i \cdot \frac{(b_0 - a_0)}{N+1}$, $i = 0, 1, \ldots, N+1$.

2. Calculate the function value at the found point: $J(u_i)$, $i = 0, 1, \ldots, N+1$.

3. Among the points u_i, $i = 0, 1, \ldots, N+1$, find one in which the function takes the smallest value: $J(u^*) = \min_{0 \leq i \leq N+1} J(u_i)$.

The inaccuracy of finding the point of minimum of brute force does not exceed $\varepsilon \leq \frac{(b_0 - a_0)}{N+1}$.